Control system Charcteritic

1- Stability
2- Time Response
3- Tracking
4- Robustness

3-Tracking

- هو قدرة وصول النظام للخرج المطلوب عند تغير القيمة المطلوبة

4-Robustness

- هو قدرة النظام على الوصول إلى نفس الخرج في ظروف مختلفة

1-Stability

- هو قدرة النظام للوصول لحالة Steady state

2- Time Response

- هو قيمة الخرج كدالة في الزمن

Routh Array (stability)

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + - - - + a_n = 0$$

$$b_0 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_1 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

s^n	a_0	a_2	a_4	-
s^{n-1}	a_1	a_3		
s^{n-2}	b_0	b_1	b_2	-
s^{n-3}	c_0	c_1	c_2	-
s^0	a_n			

- التغير في الإشارة يناظر وجود Pole يقع في R.H.S

- لو وجد عنصر في أول عمود قيمته صفر يحاط بقيمة ε وهي قيمة صغيرة تقترب من صفر (موجبة)

- لو وجد صف كله صفر تكون المعادة $A(s)$ من الصف الذي يسبقه ثم نقوم باشتقاقها

Ex: $s^3 + 2s^2 + 4s + 8 = 0$

$A(s) = 2s^2 + 8$

$\frac{dA(s)}{ds} = 4s$

s^3 +	1	4
s^2 +	2	8
	~~0~~	
s^1 +	4	
s^0	8	

Critically Stable

$s^4 - 3s^2 + 4s + 8 = 0$

s^4 +	1	-3	8
s^3 +	0	4	
s^2 −	$\frac{-3\varepsilon - 4}{\varepsilon}$	8	
s^1 +	$\frac{-3\varepsilon - 4}{\varepsilon} * 4 - 8\varepsilon$		
s^0 +	$\left(\frac{-3\varepsilon - 4}{\varepsilon}\right)$ → 8		

no. of poles of R.H.S

$n_+ = 2$

Un Stable

$s^4 + 2s^3 - 3s^2 - 8s - 4 = 0$

$A(s) = s^2 - 4$

$\frac{dA(s)}{ds} = 2s$

s^4 +	1	-3 -4
s^3 +	2	-8
s^2 +	1	-4
s^1	~~0~~	
+	2	
s^0 −	-4	

no. of poles of R.H.S

$n_+ = 1$

Un Stable

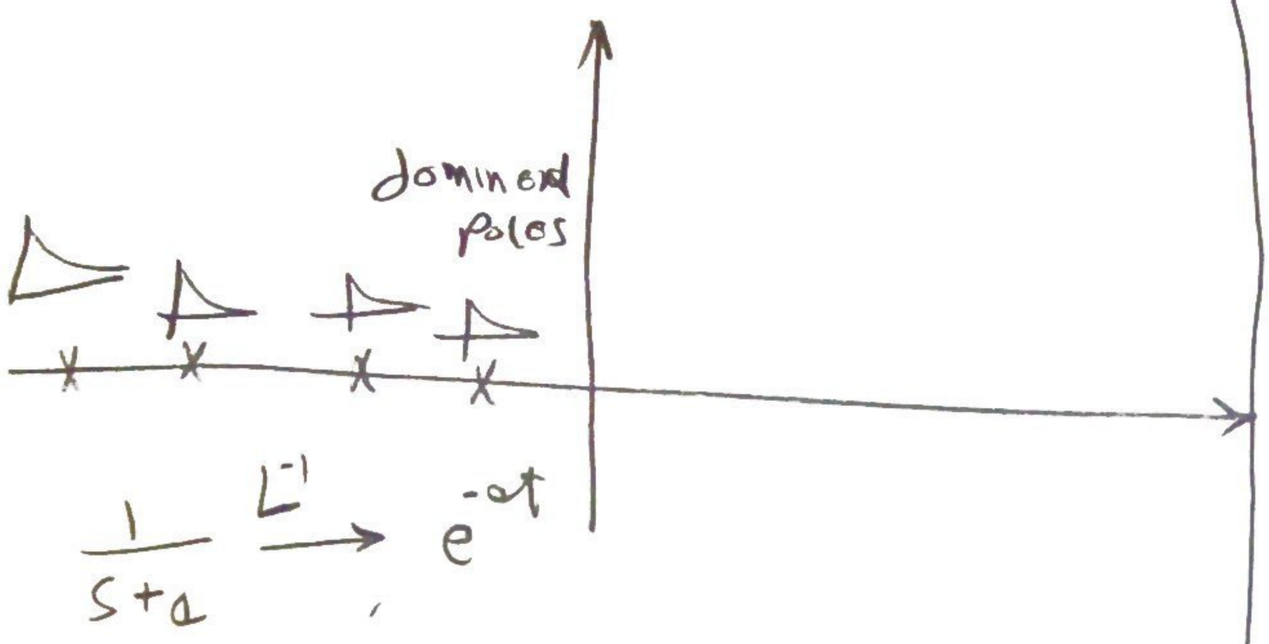
$n_- = 3$

$n_0 = 0$

$A(s) = s^2 - 4 = 0$

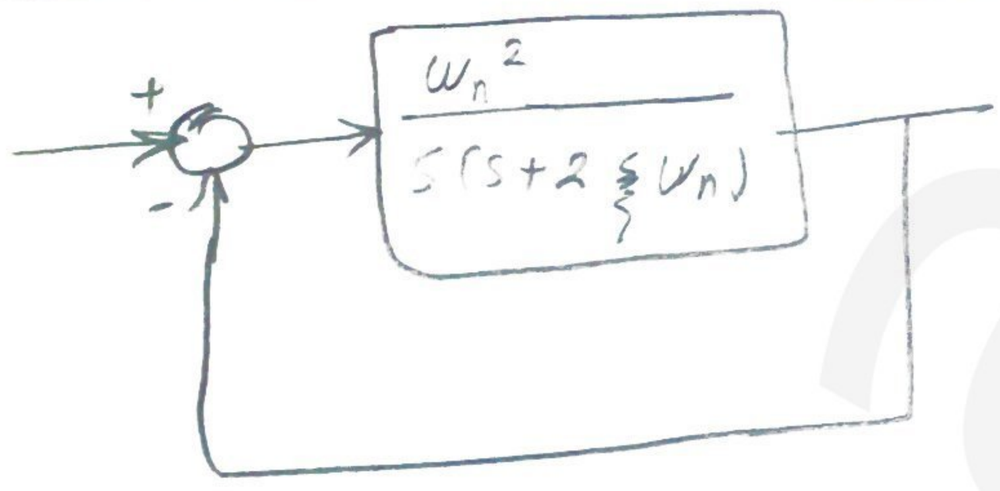
$s = \pm 2$ لاتقع على المحور

Time Response of 2nd order system



نلاحظ عندما تقترب poles من اليمين
يكون النظام أبطئ لذلك تسمى المؤثرة أكثر

عند تحويل أي نظام إلى 2nd order
يتم ذلك من خلال استخدام 2 poles على اليمين



$$G(S)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n \rightarrow$ natural frequency

$\zeta \rightarrow$ damping ratio

[1] Under damped response $0<\zeta<1$

$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

Equation $\rightarrow \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2-4\omega_n^2}}{2}$

$$s_{1,2}=-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

$$s_{1,2}=-\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$

ω_d

$\omega_d \rightarrow$ damped frequency

Step Response

$$Y(s)=\frac{1}{s}\cdot\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$y(s)=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

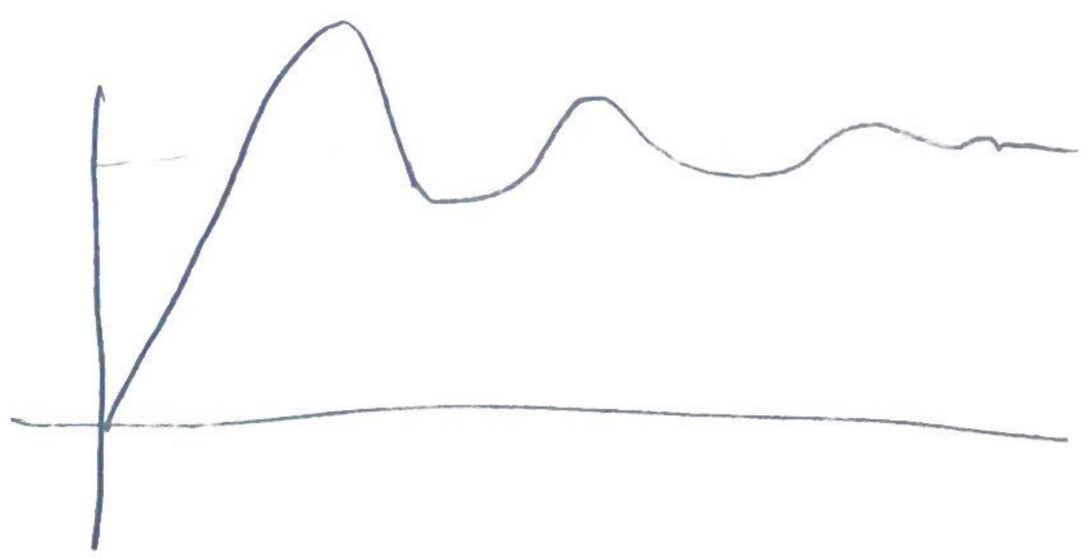
$$y(t)=1-\cos(\omega_d t)\,e^{-\zeta\omega_n t}-\frac{\zeta\omega_n}{\omega_d}\sin(\omega_d t)\,e^{-\zeta\omega_n t}$$

$$y(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\cos+\zeta\sin\right)$$

$\sin\theta$ $\quad$ $\cos\theta$

$$y(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\theta)$$

$$\theta=\cos^{-1}\zeta$$



(2) Critically damped $\xi = 1$

$S_1, S_2 = -\omega_n$

$$G(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{\omega_n}{s+\omega_n}$$

$$y(t) = 1 - \omega_n t e^{-\omega_n t} - \omega_n e^{-\omega_n t}$$

(3) over damping $\xi > 1$

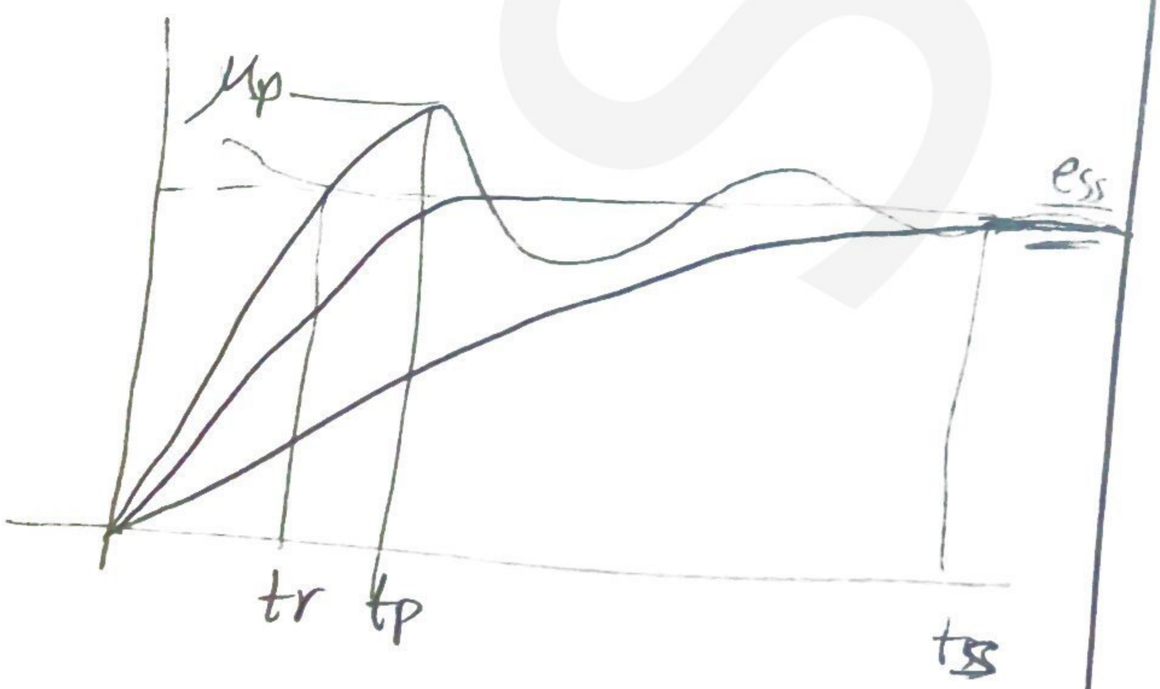
$$S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+a)(s+b)}$$

$$Y(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+a)(s+b)}$$

$$Y(s) = \frac{1}{s} - \frac{c_1}{s+a} - \frac{c_2}{s+b}$$

$$y(t) = 1 - c_1 e^{-at} - c_2 e^{-bt}$$



$$t_r = \frac{\pi - \theta}{\omega_d} \qquad \theta = \cos^{-1}\xi$$

$$t_p = \frac{\pi}{\omega_d}$$

$$t_s = \frac{4}{\xi\omega_n} \rightarrow \text{2\% error}$$

$$t_s = \frac{3}{\xi\omega_n} \rightarrow \text{5\% error}$$

$$\mu_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = \frac{y(t_p) - y(\infty)}{y(\infty)} * 100$$

Control

سكشن 2 م. شهاب الخميس 8/10/2015

Control System characteristics :-

- Stability
- Time Response
- Tracking
- Robustness

* Stability
# Routh method

$a s^n + a_1 s^{n-1} + \cdots + a_n = 0$

a_0	a_2	---
a_1	a_3	---
b_1	b_2	--
c_1	c_2	--

Two special cases

① Zero Element

Zero $\rightarrow \varepsilon$ (small positive value)

② Zero row

Auxillary eqn $P(s)$

$\frac{d}{ds} P(s)$

Ex: $s^4 + 2s^3 - 3s^2 - 8s - 4 = 0$

Solution

s^4	1	-3 -4
s^3	2	-8
s^2	1	-4
s^1	2	
s^0	-4	

$s^2 - 4$

$\frac{d}{ds} P(s) = 2s$

from Aux $P(s)$

$P = P_- + P_0 + P_+$

$1 \quad +0 \quad +1$

$n = n_+ + n_0 + n_-$

$1 + 0 + 1$

2

$\Rightarrow \underset{3}{n_-} \quad \underset{0}{n_0} \quad \underset{1}{n_+}$

Ex: $s^4 - 3s^2 + 4s + 8 = 0$

Solution

s^4	1	-3 8
s^3	ε	4
s^2	$\frac{-3\varepsilon - 4}{\varepsilon}$	8
s^1	c_1	
s^0	8	

$$c_1 = \frac{\left(\frac{-3\varepsilon - 4}{\varepsilon}\right)4 - 8\varepsilon}{\left(\frac{-3\varepsilon - 4}{\varepsilon}\right)}$$

$$= \frac{-12\varepsilon - 16 - 8\varepsilon}{-3\varepsilon - 4}$$

n_+	+	n_0	+	n_-	
2	+	0	+	(2)	تغيير الإشارة ترتيب

Ex: $S^3 + 2S^2 + 4S + 8$

S^3	1	4
S^2	2	8 ←
S^1	4	
S^0	8	

→ Aux eqn : $2S^2 + 8 = 0$

$\frac{d}{ds}$ P → $4S$

Aux eqn

$P = P_+ + P_0 + P_- = 2$

(P_+ ← 0, P_0 ← 2, P_- ← 0)

for ch. eqn

$n = n_+ + n_0 + n_- = 3$

(n_+ ← 0, n_0 ← 2, n_- ← 1)

from Aux: 0 → (2), x 1

n_+	n_0	n_-
↓ 0	↓ 2	↓ 1

Time Response of 2nd order system

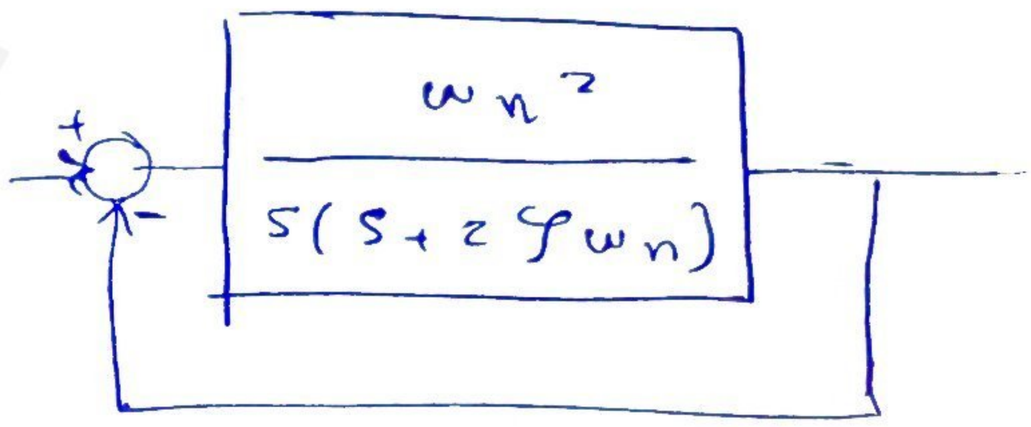
Any system has

ω_n → Natural frequency

ζ → damping ratio

* dominant poles are responsible for system's Dynamics.

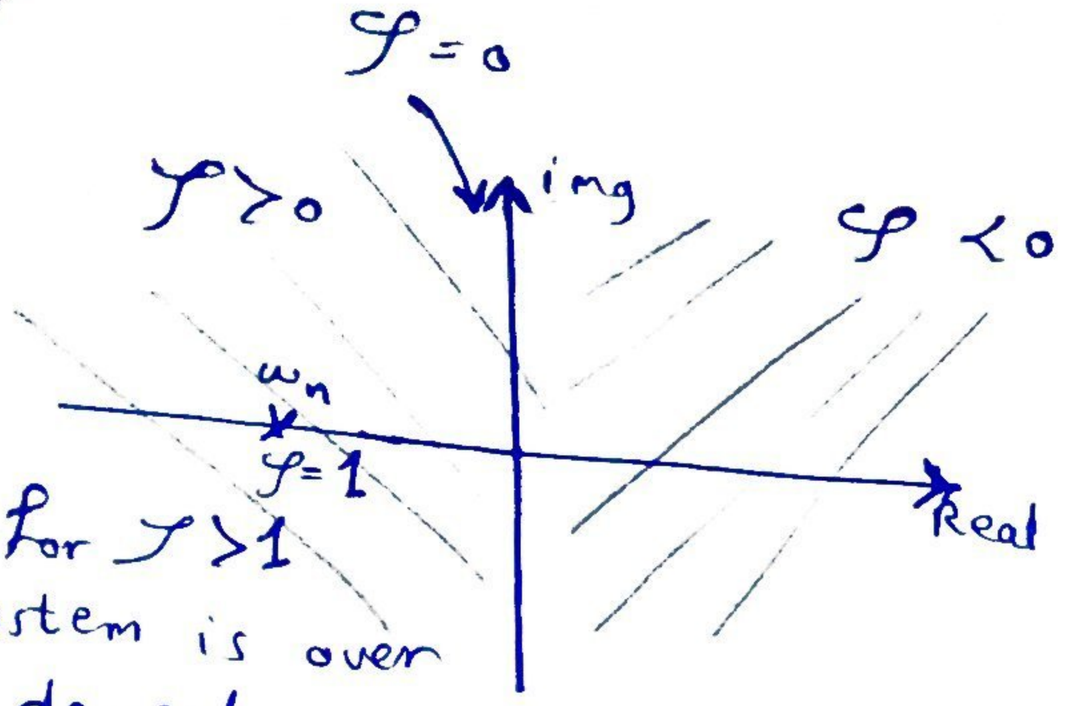
usually we get the highest pole because it has more transient time.



$$\frac{Y(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$$S_{1,2} = \frac{2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$



* for $\zeta > 1$ system is over damped
* for $1 > \zeta > 0$ ⇒ the system is underdamped
* for $\zeta = 1$ ⇒ the system is critically damped

$$S_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$= -\underbrace{\zeta\omega_n}_{\sigma} \pm j\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$$

for $0 < \zeta < 1$

ω_d = damped frequency

$$\frac{Y}{R} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$r(t) = 1 \rightarrow R(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{s + \zeta\omega_n}{\text{DEN}} - \frac{\zeta\omega_n}{\text{Den}}$$

$$\therefore y(t) = 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t$$

$$= \frac{1 - e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} * \left(\sqrt{1-\zeta^2}\cos\omega_d t + \zeta\sin\omega_d t\right)$$

($\sin\theta$, $\cos\theta$)

$$\sin\omega_d t + \theta$$

$$= \frac{1 - e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$$

Case 2:

$\zeta = 1$

$S_{1,2} = -\omega_n$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$Y(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{\omega_n}{s+\omega_n}$$

$$y(t) = 1 - \omega_n t e^{-\omega_n t} - \omega_n e^{-\omega_n t}$$